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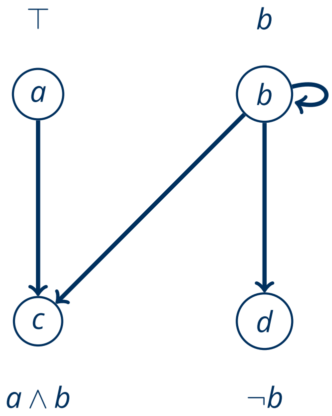
ADF-BDD.DEV: Insights to undecided Statements in Abstract Dialectical Frameworks

AIˆ3 2023 // Rome, Italy, November 9, 2023

Abstract Dialectical Frameworks [BES+18]



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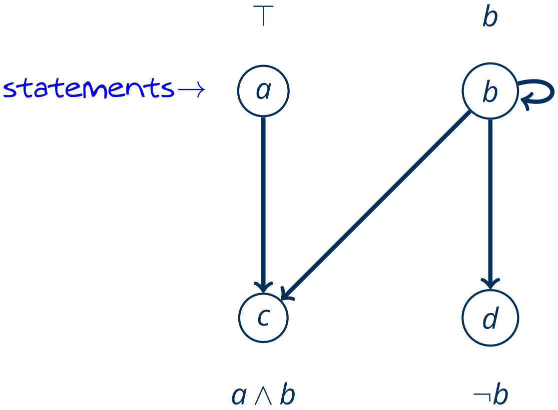

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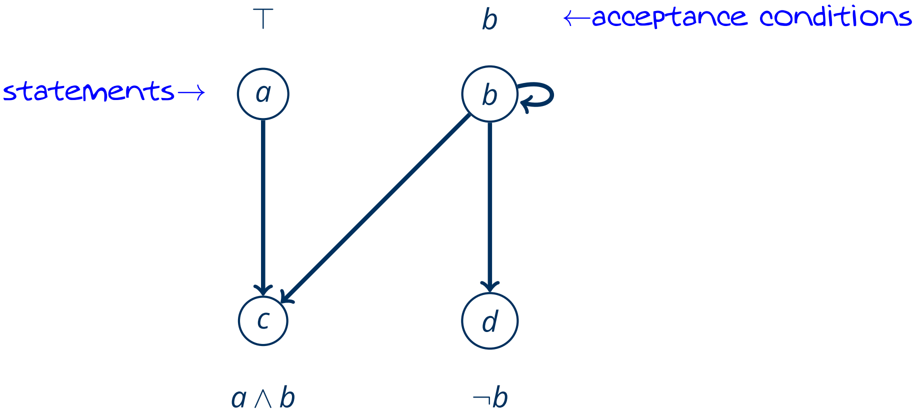

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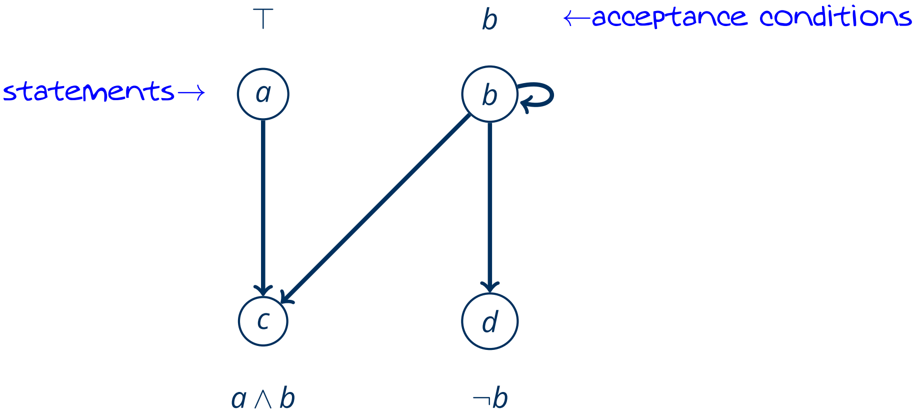
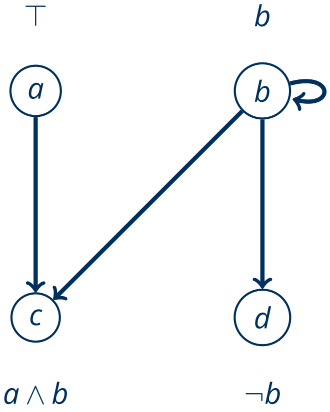


semantics: $\Gamma_D : \mathcal{V}^3 \mapsto \mathcal{V}^3$

Abstract Dialectical Frameworks [BES+18]

$\leftarrow$acceptance conditions

statements$\rightarrow$



$a \mapsto \boldsymbol{u}, b \mapsto \boldsymbol{u}, c \mapsto \boldsymbol{u}, d \mapsto \boldsymbol{u}$

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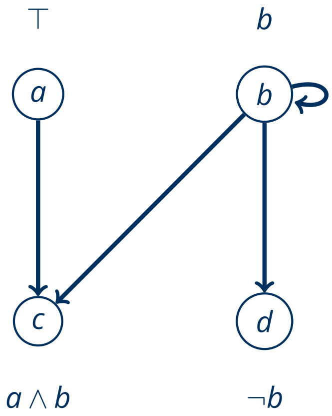

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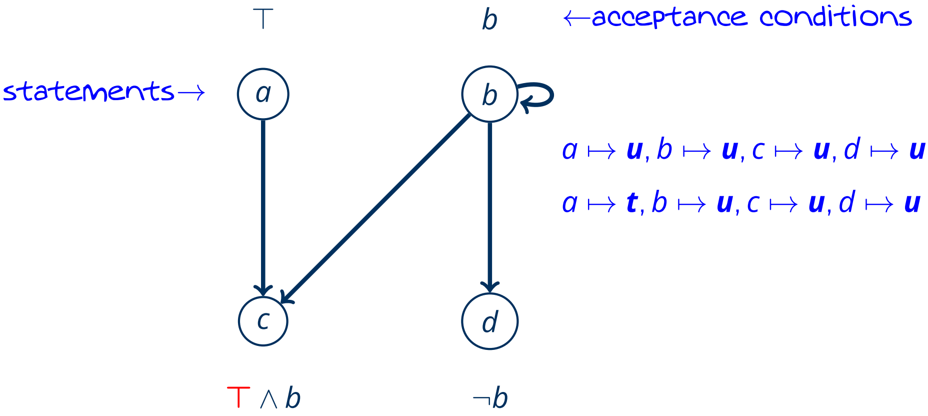

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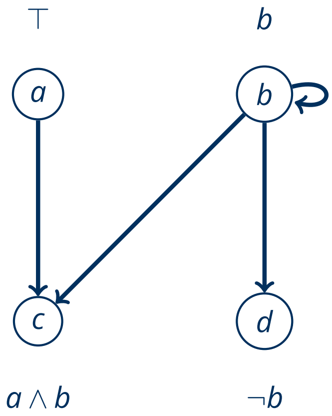


grounded, complete

$a \mapsto \boldsymbol{t}, b \mapsto \boldsymbol{u}, c \mapsto \boldsymbol{u}, d \mapsto \boldsymbol{u}$

complete

$a \mapsto \boldsymbol{t}, b \mapsto \boldsymbol{t}, c \mapsto \boldsymbol{t}, d \mapsto \boldsymbol{f}$

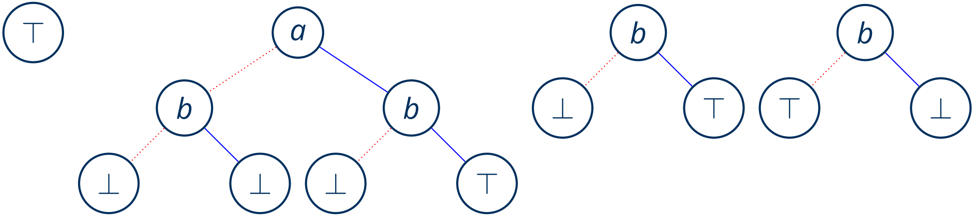
complete, stable

$a \mapsto \boldsymbol{t}, b \mapsto \boldsymbol{f}, c \mapsto \boldsymbol{f}, d \mapsto \boldsymbol{t}$

semantics: $\Gamma_D : \mathcal{V}^3 \mapsto \mathcal{V}^3$

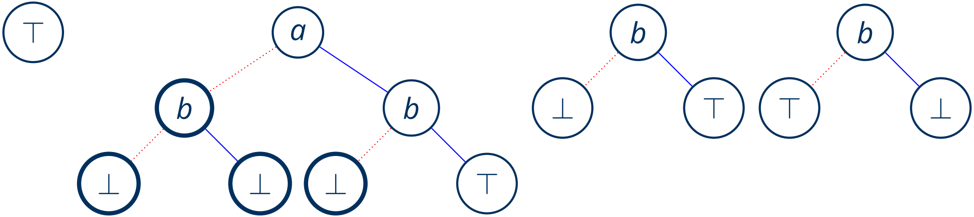
ordered Binary Decision Tree

- Tree: inner nodes are variables and leafs are truth constants $\top$ and $\bot$
- Inner node has lo and hi child
- Every path from root to leaf needs to follow pre-defined strict ordering of variables



reduced ordered Binary Decision Diagram [Bry86]

- $lo(n) = hi(n)$, then replace n by $hi(n)$
- if $n = v$, then replace v by n globally (violate tree-property)
- Given a variable order, this representation is unique under logical equivalence of formulae



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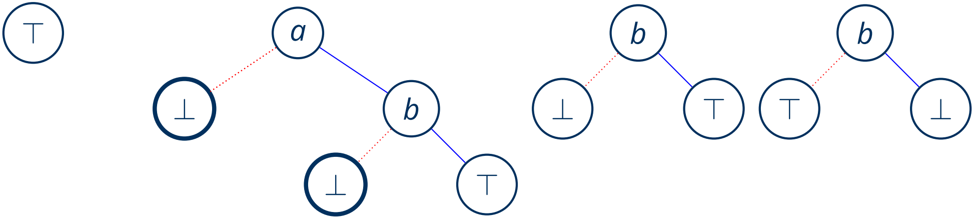

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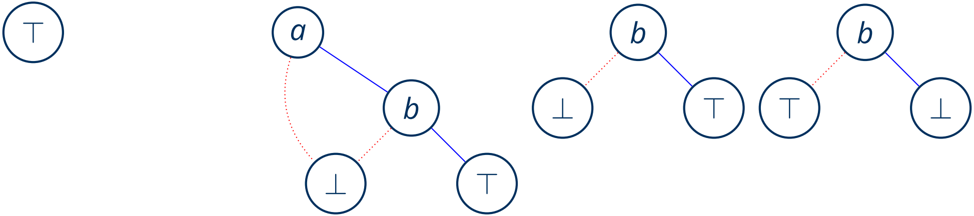

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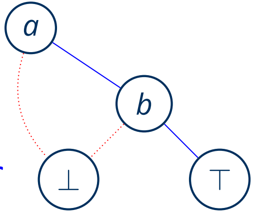
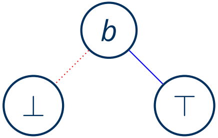
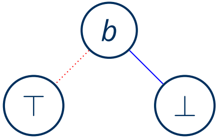




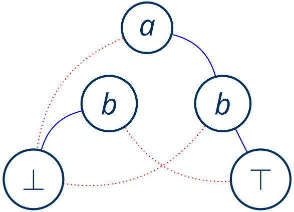


- Restriction linear

- Optimal variable order in NP
- Check for (un-)SAT and TAUT constant [DM02]

New idea: roBDDs to represent ADFs [EGRW22b]

- To each statement, one BDD is related as the acceptance condition
- More compact representation due to "merging" of nodes



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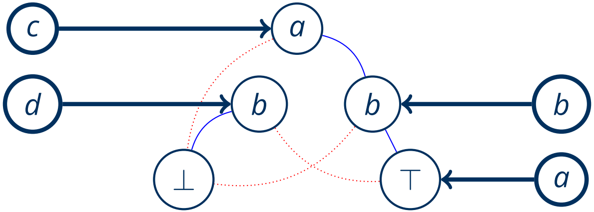


roBDDs to represent ADFs

Theorem

Given the BDD representation of an ADF D, the result of applying Γ_D to any three-valued interpretation $\mathcal{I}$ can be computed in polynomial time.

Theorem

Given an ADF D in BDD representation, there is a polynomial algorithm that computes the grounded interpretation of D.

Corollary

Verifying whether a three-valued interpretation is a model or is stable in an ADF represented by BDDs is in P. Moreover credulous reasoning is in NP and sceptical reasoning in coNP.

ADF-BDD solver [EGRW22a]

- Written in `Rust`
- BDDs
  - own implementation
  - biodivine-bdd for faster instantiation
- Various BDD-modes (own, biodivine, hybrid)
- Grounded, complete, and stable semantics
- Github, Library, and Binary available
  - hub: `https://github.com/ellmau/adf-obdd`
  - lib: `https://crates.io/crates/adf-bdd`
  - bin: `https://crates.io/crates/adf-bdd-bin`

ADF-BDD.dev

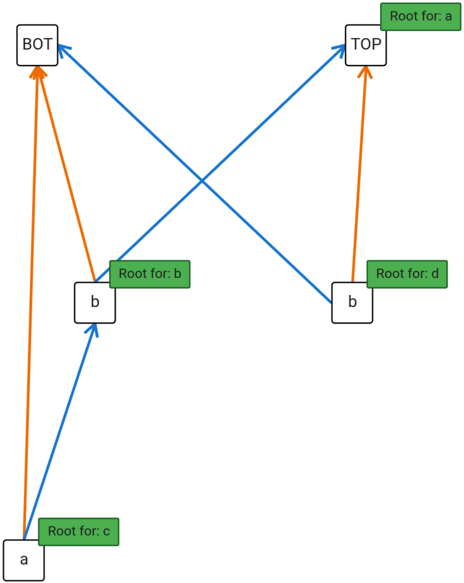
- Web-application for adf-bdd
- Visualisation of results as BDDs
  - Parsed state
  - Every semantics model
- G6 graph visualisation library used [WBL$^+$21]
  - Dagre algorithm for graph representation
  - Ranks nodes into hierarchy
  - Minimises number of crossing edges
- Colour-coded BDD
  - Orange is lo
  - Blue is hi
  - Green labels are the roots for each statement

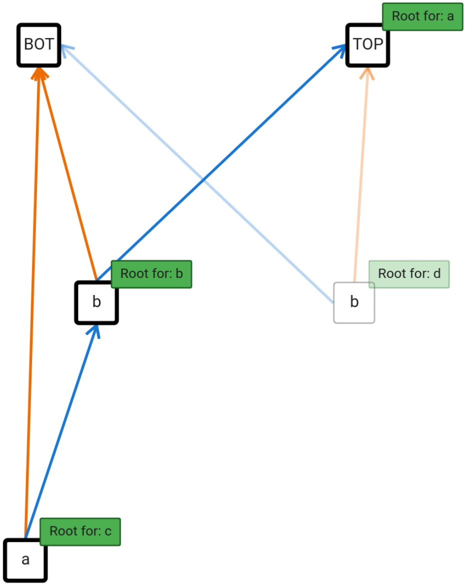
ADF-BDD.dev
Visualisation Insights

- ***t*** and ***f*** already decided by fixpoints
- Reason for each ***u*** is given by corresponding BDD
- Focus onto sub-diagrams
- No redundant links, only variables that have an impact

ADF-BDD.dev

Online presentation of ADF-BDD.dev or further slides


Root for: a
BOT
TOP
Root for: b
b
Root for: d
b
Root for: c
a



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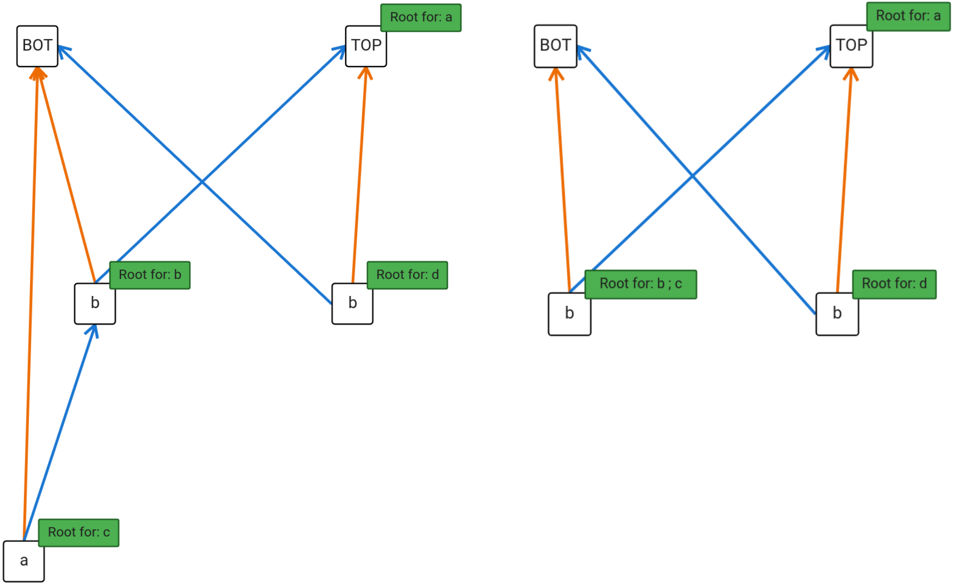
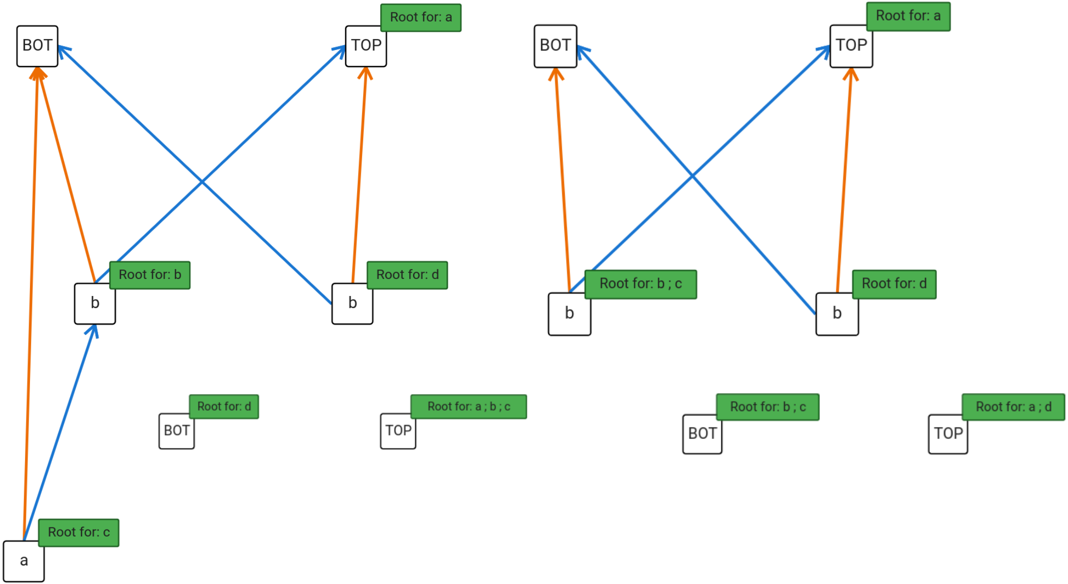


BOT
TOP
Root for: a
b
Root for: b
b
Root for: d
a
Root for: c
BOT
TOP
Root for: a
b
Root for: b ; c
b
Root for: d


BOT
TOP
Root for: a
b
Root for: b
b
Root for: d
Root for: d
BOT
TOP
Root for: a ; b ; c
a
Root for: c
BOT
TOP
Root for: a
b
Root for: b ; c
b
Root for: d
BOT
Root for: b ; c
TOP
Root for: a ; d

Future Work

- Direct editing of BDDs
  - ADF-Design
  - Enforcement
- Import and export of BDDs
- More assistance for non-familiar users
- More analysis options for experienced users

Thank you for your interest!

Visit ADF-BDD at

https://adf-bdd.dev

https://ellmau.github.io/adf-obdd/

Abstract Dialectical Frameworks

Definition (Abstract Dialectical Framework (ADF))

An ADF is a tuple $\langle S, C \rangle$, where

- S is a fixed finite set of statements and
- $C := \{\varphi_s\}_{s \in S}$ is a set acceptance conditions for statements, which corresponds to propositional formulae whose variable signature is S.

Abstract Dialectical Frameworks

Definition (Γ_D)

Let $D = \langle S, C \rangle$ be an ADF, $\mathcal{I} : S \mapsto \{\boldsymbol{t}, \boldsymbol{f}, \boldsymbol{u}\}$ be a three-valued interpretation, and $\Gamma_D(\mathcal{I}) : S \mapsto \{\boldsymbol{t}, \boldsymbol{f}, \boldsymbol{u}\}$ with

$$s = \begin{cases} \mathbf{t} & \text{if } \models \varphi_s(\mathcal{I}); \\ \mathbf{f} & \text{if } \varphi_s(\mathcal{I}) \models \bot; \\ \mathbf{u} & \text{otherwise.} \end{cases}$$

Definition (Semantics)

$\mathcal{I}$ is

- complete if $\mathcal{I} = \Gamma_D(\mathcal{I})$
- grounded if $\mathcal{I} = \mathit{lfp}(\Gamma_D)$

Abstract Dialectical Frameworks

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- stable if $\mathcal{I}$ is two-valued, complete, and the for the grounded interpretation $\mathcal{W}$ of $D^{\mathcal{I}}$ it holds that $\mathcal{I}(s) = \boldsymbol{t}$ implies $\mathcal{W}(s) = \boldsymbol{t}$

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Definition (Reduction)

Let $D = \langle S, C \rangle$ be an ADF, $\mathcal{I} : S \mapsto \{\boldsymbol{t}, \boldsymbol{f}\}$ be a two-valued interpretation.
$D^{\mathcal{I}} = \langle S^{\mathcal{I}}, C^{\mathcal{I}} \rangle$, where

- $S^{\mathcal{I}} = \{s \in S \mid \mathcal{I}(s) = \boldsymbol{t}\}$
- $C^{\mathcal{I}} = \{\varphi_s[s'/\bot : \mathcal{I}(s') = \boldsymbol{f}]\}$

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